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TANGENTIAL FRICTION ON THE WALL OF A CONTAINER WITH A SPHERICAL CHARGE AND FLUID MOTION THROUGH THE CHARGE

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Viscous tangential stress on the interior wall of a container with a spherical charge, that develops in filtration of an incompressible fluid through the charge is investigated. Based on an analysis of an experimental dependence of the dimensionless stress on the Reynolds number, two critical Reynolds numbers are determined: the first number corresponds to the beginning of an abrupt drop in the stress, and the second number, to its reaching a regime that is self-similar in velocity. Comparison with the theory permits interpretation of the effects of pseudoturbulence and turbulence, respectively.

Processes that occur in granular media are complex; therefore, many essential details of hydrodynamics and transfer remain little studied. The hydrodynamic features of processes in the wall zone directly adjacent to the heat-exchange surface have practically not been investigated [1].

We study the properties of the viscous tangential stress on the interior wall of a container with a spherical charge that develops with fluid motion through the charge. Elements similar to the studied ones in geometry and occurring hydrodynamic processes are widely used in a variety of apparatuses of modern technology: filters, heat exchangers, and catalytic chemical reactors. The mechanism of the effect of by-passing of a part of the flow through the wall zones in those devices [1] is, as is expected, associated with friction properties on the interior walls. The obscure causes of the high level of by-passing make it important to study friction properties.

An experimental study of tangential stress involves great methodological difficulties, by virtue of which the number of works on this problem is small.

In [2], stress was determined by an electrodiffusion method as a function of the rate of filtering through the charge in the range of the Reynolds numbers of from 0 to 170 (the Reynolds number is constructed from the sphere diameter and the filtering rate of the fluid). The fluid is an electrolyte, $\rho = 1000$ kg/m^3, $\mu = 0.001$ kg/m·sec. The container is a tube with diameter $D = 13.8$ mm; the spheres are of two sizes: $d = 1.07$ and 3.2 mm. We used sensors of the dimensions, much larger and smaller than the diameter of the sphere, that yielded readings, coincident within 5%, with good reproducibility in multiple repackings of the charge.

In [3], the friction was found from the slope of the velocity profile near the wall in a cell of cubic packing of spheres using a laser Doppler anemometer in the range Re = 800–2900. Measurements are performed at several points along and transverse to (at the maximum cross-section) the longitudinal axis of symmetry of the cell. An immersion liquid – $\rho = 1300$ kg/m^3, $\mu = 0.0013$ kg/m·sec – was used. The packing is nine spheres of $d = 18.3$ mm in a row.

We represent the data on the tangential stress on the container wall τ_* as a function of the rate of liquid filtering through the charge U_m [2] in dimensionless form: $U'(0) = \tau_* d/\mu U_m$ versus $\mathrm{Re} = \rho U_m d/\mu$ (the solid lines in Fig. 1). Unlike the dimensional stress, which grows monotonically with the flow rate, the dimensionless stress is characterized by a more complex dependence on the number Re. On the experimental curves, we can distinguish three characteristic regions: 1-2, 2-3, and 3-4. The first region 1-2 is distinguished by an increase in the stress as a function of Re, the second region 2-3 corresponds to an abrupt drop in the observed dimensionless stress on the wall with a gradual decrease in the rate of the drop. The beginning of the drop corresponds to point 2 and is a

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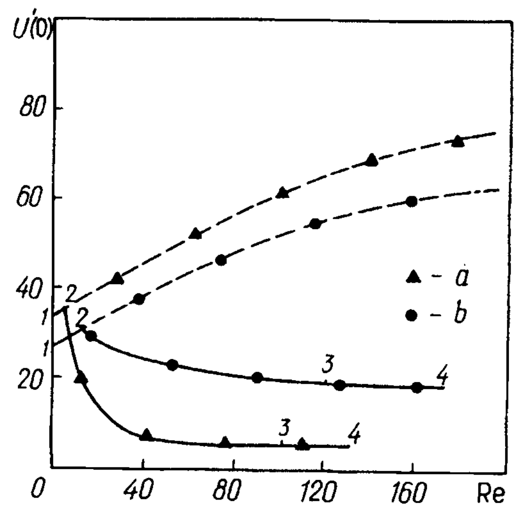

Fig. 1. Dimensionless tangential stress on a wall $U'(0)$ vs. the Reynolds number Re. The solid line is experiment, the dashed line is the theory with no allowance made for the contributions of pseudoturbulence and turbulence; a) and b) are charge spheres with diameters of 1.07 and 3.2 mm.

salient point. The third region 3-4 is distinguished by the self-similarity of stress in the Reynolds number (it begins from point 3). Point 3 cannot be distinguished on the curve as clearly as 2, since the curve degenerates asymptotically into a hozizontal straight line. We determine the position of point 3 in Fig. 1 by the condition of its 5% deviation from asymptote.

We determine two critical Reynolds numbers: the first number corresponds to the beginning of the abrupt drop in the frictional stress, and the second one, to the beginning of self-similarity. The plots show that flow in the charge of larger spheres is characterized by larger first and second critical Re numbers.

The data of [3] cannot be processed in terms of the filtering rate without averaging the tangential stress over the entire solid wall adjacent to the cell of the packing. The magnitudes of the stresses turn out to be dependent on the position of the measurement point with respect to the cell. The maximum stress is recorded on the wall in the diffuser cross-section on the axis of symmetry of the cell, and the minimum stress (it is negative), in the sphere afterpart, in the region of return flows. Data on the dimensionless stress as a function of Re were obtained by processing the results of [3] and are given in Table 1. Points 1-4 are located along the cell's axis of symmetry, respectively, in the minimum, diffuser, maximum, and confuser cross-sections of the cell, and points 5-7, on the axis that is perpendicular to the axis of symmetry of the cell in the maximum cross-section with lateral shifts from the axis of symmetry of 3.6 and 9 mm. From the data of Table 1, we can establish that, despite the two- to threefold change in the number Re, the dimensionless stress is approximately the same at the characteristic points; consequently, the average stress will be the same, too. This qualitative result enables us to conclude that there exists at large Re numbers self-similarity of the tangential stress with respect to Re in cubic packings of spheres, too.

When an electrodiffusion stress meter was used [2], averaging was apparently performed by the device itself.

We move on to a theoretical analysis of the properties of tangential stress on the wall of a container with a spherical charge and an incompressible viscous fluid moving through it.

An essential factor retarding a theoretical study of the problem is the absence of a universally adopted and reliably substantiated closed equation of filtration that enables us to allow for the condition of adhesion on a solid wall. We know the Brinkman equation [4]:

$$dP/dx = -\mu/kU + \mu d^2U/dy^2 . \tag{1}$$

TABLE 1. Dimensionless Viscous Tangential Stress on a Wall $U'(0)$ as a Function of the Number Re for Different Points of Measuring in a Wall Cell of a Cubic Packing of Spheres

Re	$U'(0)$						
	1	2	3	4	5	6	7
830	148	162	148	148	32	−21	−15
1670	113	224	149	159	26	−21	−18
2900	109	197	157	143	22	−15	−16

The x axis is directed along the wall downstream; the y axis is perpendicular to the wall and directed into the charge; the coordinate origin is located on the wall. Equation (1) enables us to allow for the adhesion condition, but it is suitable only for small filtering rates and, furthermore, as has been shown in [1], should contain μ_{eff} rather than μ in the last term of the right-hand side, the estimate for μ_{eff} being known only for media with a porosity higher than 0.7-0.8. In [5], by the method of averaging over the volume, an open equation of filtering that is suitable for high filtering rates is obtained:

$$dP/dx = -\mu/kU + \mu d^2U/dy^2 - q\rho U^2 + \rho d\,(\langle A_xA_y\rangle)/dy. \tag{2}$$

The fourth term on the right-hand side of (2) allows for the transverse transfer of momentum by pseudoturbulence: by fluctuations of the velocity field of a fluid that moves through an irregular structure. In [6], it was proposed to also take into account the contribution of turbulence in (2) by a term of the form $\rho d\langle u'v'\rangle/dy$ on the right-hand side. Allowance for pseudoturbulence and turbulence separately makes sense, since both effects differ in length scale: for the first effect, on the order of the grain diameter [7]; for the second, much smaller than the pore size [8]. They also have different causes: the development of turbulence is associated with a loss of dynamic stability of flow in charge pores [8] while pseudoturbulence is due to macroheterogeneities of the properties and structure of the material that affect fluid motion in the pores, bend current lines, and induce fluctuations of the velocity field and their related transfer [7].

We calculate the magnitude of the tangential stress on the wall of the tube containing the charge based on Eq. (2) as a more general equation in view of the equalities $k = d^2/a_1$ and $q = a_2/d$, where a_1 and a_2 are quantities dependent only on the porosity of the charge. To simplify the calculations, we take the radius of the tube to be infinite. We denote the filtering rate at a sufficient distance from the wall (infinity) as U_m. From the experiments, it is known that the wall has an effect on flow in the charge to distances of about several grain diameters and the maximum rate on the tube axis is observed only for charges with $D/d < 3.3$ [1]; therefore, at large depths and for not very large spheres, from (3) we obtain

$$dP/dx = -(a_1\mu U_m/d^2 + a_2\rho U_m^2/d), \tag{3}$$

from which it follows that

$$\mu d^2U/dy^2 + \rho d/dy\,(\langle A_xA_y\rangle + \langle\overline{u'v'}\rangle) - a_1\mu U/d^2 - a_2\rho U^2/d + $$
$$+ a_1\mu U_m/d^2 + a_2\rho U_m^2/d = 0. \tag{4}$$

We dedimensionalize (4), taking the grain diameter as the length scale and U_m as the rate scale:

$$U'' + \mathrm{Re}\,(\langle A_xA_y\rangle + \langle\overline{u'v'}\rangle)' - a_1U - a_2\,\mathrm{Re}\,U^2 + a_1 + a_2\,\mathrm{Re} = 0. \tag{5}$$

Using the identity $U'' = (U'^2)'/(2U')$, we rearrange (5) and integrate once between the limits of from 0 to ∞, allowing for $U'(\infty) = 0$, $U(\infty) = 1$, $U(0) = 0$:

$$U'(0) = \left(a_1 + \frac{4}{3} a_2 \operatorname{Re} + 2 \operatorname{Re} \int_0^\infty (\langle A_x A_y \rangle + \overline{\langle u'v' \rangle})' \, U' dy\right)^{1/2}, \tag{6}$$

Processing the data of [2] on the rate of filtering through the charge as a function of the pressure drop enabled us to determine the coefficients $a_1 \doteq 160(1 - \varepsilon)^2/\varepsilon^3$ and $a_2 = 1.75(1 - \varepsilon)/\varepsilon^3$, in which the porosity $\varepsilon = 0.375 + 0.78\,(d/D^2)$, according to [9]; in this case, the deviation of the experimental points from the calculation does not exceed 2–5%, while the permeabilities agree well with the experimental ones. Let us compare in Fig. 1 the dependence of $U'(0)$ on Re that is obtained from Eq. (6) with no allowance made for the contribution from the integral (dashed curve) with the data of [2] (solid curve).

In region 1-2 there is complete agreement between the calculated and experimental curves. At point 2, when the number Re attains its first critical value, the curves disagree sharply: the theoretical curve keeps on growing as $\operatorname{Re}^{1/2}$ while the experimental one begins to drop.

Usually in filtration theory, disagreements of this kind are explained either by increased permeability near the wall [1] or by the influence of the deviation of the resistance law from a linear one for the charge [2]. Let us show that both these factors in the given case cannot account for the deviation observed.

The decrease in the wall stress cannot be explained by a direct influence of the increase in the permeability near the wall. Indeed, let the permeability be increased near the wall and be characterized by the law $a_1(1 - f(y))$, in which $f(y)$ is an arbitrary function decreasing monotonically to zero at infinity. Substituting it into (5), after rearrangements we obtain in view of $f'(y) < 0$ on the right-hand side of (6) the additional term $(-a_1 \int_0^\infty U^2 f' dy) > 0$, i.e., the stress can only grow.

The beginning of the drop in the frictional stress on the wall is associated with the appearance of the quadratic-in-velocity component of the resistance in the equation of filtration only to the extent that both phenomena are quadratic in velocity. Indeed, attainment by the ratio of the magnitudes of the quadratic and linear components in (3) of a preset limit, for example, the number const $= (a_2 \rho U_m^2/d)/(a_1 \mu U_m/d^2) = \operatorname{Re}/(1 - \varepsilon)$, can be considered as the beginning of the deviation of the resistance law. It is found that the lower the porosity of the charge, the larger the characteristic number $\operatorname{Re} = \text{const}(1 - \varepsilon)$ of the beginning of the deviation of the resistance law from a linear one. The dependence of the first critical Re number on porosity, as the plot shows, is inverse: for a charge with 3.2 mm spheres, $\varepsilon = 0.407$, Re = 12–13; for 1.07 mm spheres, $\varepsilon = 0.380$, Re = 3–4, i.e., the higher the porosity and, correspondingly, the straighter the pore channel, the larger the first critical Re number. The rate of the drop in the tangential stress as a function of Re after the first critical number also turns out to be higher for a denser charge.

According to Eq. (6), the disagreement between the experimental and calculated curves is due to neglect of the influence of the integral term and the effects of pseudoturbulence and turbulence. The influence of turbulence cannot induce the observed drop in the friction in the range after the first critical Reynolds number for two reasons. First, according to the data of visualization and anemometry, pulsations over the entire cross section of the charge appear for Re > 34 [1], and it is only for Re > 100–120 [8, 10] that turbulent transfer becomes significant. Second, from experiments [8] it is known that turbulence in the charge enhances the processes of mixing and transverse momentum transfer, increases the curvature of the velocity profile near the wall, and should necessarily lead to growth in the tangential stress. These facts enable us to uniquely interpret the effect of the drop in the tangential stress on the wall as a result of transverse pseudoturbulence momentum transfer being involved in the process. In the charges that are characterized by more-crooked pore channels (lower porosity), it is natural to expect a higher level of pseudoturbulence and, in accordance with the experiments, an earlier onset of and a more abrupt drop in the stress. As the region of high Reynolds numbers is approached the turbulence that enhances mixing and friction begins to gradually get involved in the process of transverse momentum exchange. Pseudoturbulence and turbulence mechanisms begin to compete, tending to decrease and to increase, respectively, the wall friction. This leads to a gradual slowing down of the rate of the drop in the dimensionless tangential stress and its reaching a regime that is self-similar in Re for Re > 100–120 for both dimensions of the spheres, which correlates with data [8, 10] on the beginning of a developed turbulent regime of filtration. According to Fig. 1, the second critical Reynolds number is 100 for 1.07 mm spheres and 120 for 3.2 mm spheres.

The existence of a region of self-similarity of tangential stress 3-4 has been established experimentally up to Re = 170; however, taking into account the data of Table 1, this region can turn out to be much wider.

The experimentally observed drop in the dimensionless stress on the wall that coincides in the geometry of the given problem with the first derivative of velocity with respect to the transverse coordinate should lead, for a constant filtering rate away from the wall, to a significant widening of the region occupied by the boundary layer. This phenomenon is indeed observed when the effect of by-passing of the liquid through wall parts of the charge is experimentally studied. The experiments are usually performed in the range after the first critical Reynolds number and it turns out that the width of the region of by-passing attains $3-5d$ [1]. This is an abnormally large width to be explained only by a near-wall increase in porosity or permeability.

The obtained results point to the necessity of allowance for the influence of pseudoturbulence and turbulence in studying transfer processes in charges near heat exchange surfaces.

The practical significance of the present work consists in establishing and determining the magnitude of two critical Reynolds numbers based on analysis of an experimental dependence of the dimensionless stress on the Reynolds number. The first number corresponds to the beginning of an abrupt drop in the stress, and the second number, to its reaching a regime that is self-similar in velocity. In the range up to the first critical number Re, the Brinkman equation can be used for calculating the tangential stress on the wall, while for higher Re numbers, equations such as (2) and (4) can be used.

NOTATION

x, longitudinal coordinate; y, transverse coordinate; P, pressure; ρ, density; μ, μ_{eff}, coefficient and effective coefficient of dynamic viscosity of liquid; d, sphere diameter; D, tube diameter; U, filtering rate; U_{m}, rate of filtering away from a wall; Re, Reynolds number; τ_*, viscous tangential stress on wall; $U'(0)$, dimensionless stress; k, permeability; ε, porosity of charge; a_1, a_2, q, constants; $\langle A_x A_y \rangle$, $\langle \overline{u'v'} \rangle$, pseudoturbulent and turbulent components of transverse momentum transfer.

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